# Study of Predictive Control <br> for Permanent Magnet Synchronous Motor Drives 

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#### Abstract

The paper deals with a study of model predictive control applied to three-phase Permanent Magnet Synchronous Motors (PMSM). These motors are used in drives of robots and machine tools. The construction of their model is discussed here with respect to a model-based control design. The model is composed via mathematical-physical analysis. The analysis is outlined in the main theoretical points. The predictive control is explained as a promising alternative to standard solution based on vector cascade control. In predictive control design, the quadratic criterion, equations of predictions and specific square-root optimization procedure are explained. The proposed solution is illustrated by several simulation examples and compared with data records of real experiment of vector control.


## I. INTRODUCTION

Synchronous motors with a three-phase stator winding and a rotor with permanent magnets (Alternate Current - AC motors) belong to the last, up-to-date generation of motors. They are applied as drives to the machine tools and robots. Unlike Direct Current (DC) /brushes/ motors and Electrically Commuted (EC) /DC brushless/ motors, Permanent Magnet Synchronous Motors (PMSM) (Fig. 1) may be configured as linear motors. They work on principle of simultaneous control of amplitude and frequency of all three terminal harmonic currents with Pulse-Width-Modulation (PWM). The stator of a three-phase AC motor represents three sinusoidally distributed windings whose axes are displaced by $120^{\circ}$. When the windings are excited with balanced threephase sinusoidal currents, the combined effect is equivalent to having a single sinusoidally distributed winding excited with a constant current and rotating at the stator frequency. Magnetic field of the rotor is supplied by permanent magnets instead of electromagnets [5].

From control point of view, there are three main tasks: position control, speed control and current (torque) control. The tasks are closely related to a control configuration or control loops. An outer loop is the position loop, a middle loop is the speed loop and an internal loop is the current loop.

[^0]In this paper, the speed control task is studied. Thus, the speed and current loops will be investigated. The task will be discussed for the conventional control approach based on vector control with a cascade of PI controllers and for advanced control approach based on Generalized Predictive Control (GPC) [1], [4], [5]. The GPC is investigated as a general, flexible alternative, which solves the both speed and current loops together.

The paper is organized as follows. The section II deals with a suitable mathematical-physical model for control design. The section III briefly describes conventional loop scheme of vector control. The section IV discusses the model modification and related assumptions for predictive control design. The section V concerns with the main points of GPC design. In the section, there is a derivation of equations of predictions and explanation of square-root minimizing procedure of quadratic criterion. The generation of control actions as a result of the minimization is discussed. The section VI demonstrates the behaviour of the conventional vector control and the behaviour of the model predictive control.


Figure 1. Schematic cross section of PM Synchornous Motor with pole pair number $\mathrm{p}=3$ and pole number $\mathrm{pp}=6(=2 \mathrm{p})$

## II. Control-Oriented Model of PMSM Drives

Mathematical-physical model of PMSM drives is important both for the outline of conventional vector control [3], [5] and mainly for model-based control approaches in general. The model serves as a simulation model for rapid prototyping of the controllers. The model of permanent magnet synchronous motors arises from several natural laws and relations. Note, that the focus is given on stator part of the motor, where the electric winding (coils) are built in. From rotor point of view, only knowledge of magnetic properties of permanent magnets is necessary.

## A. Used Notation

The model covers the relations of the equilibrium of current and voltage and appropriate relations of voltage distribution for individual phases of the three-phase system. The model contains number of parameters. Their notation and appropriate units are given as follows:
$R_{S}$ - stator resistance [ $\Omega, \mathrm{Ohm}$ ]
$L_{S}$ - stator inductance [H, Henry]
$\psi_{M}$ - rotor magnetic flux [Wb, Weber]
$p$ - number of pole pairs, $\mathrm{pp}=2 \mathrm{p}$ - pole number
$B \quad$ - viscous coefficient of the load $\left[\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-1}\right]$
$J$ - moment of load inertia $\left[\mathrm{kg} \mathrm{m}^{2}\right]$
$I_{S} \quad$ - supply current [A]
$U_{S}$ - supply voltage [V]
$i_{S A}, i_{S B}, i_{S C}$ - currents of individual phases $A, B, C$ [A]
$u_{S A}, u_{S B}, u_{S C}$ - voltages of individual phases $A, B, C[\mathrm{~V}]$
$i_{S \alpha}, i_{S \beta}$ - currents in $\alpha-\beta$ system [A]
$u_{S \alpha}, u_{s \beta}$ - voltages in $\alpha-\beta$ system [V]
$i_{s d}, i_{S q}$ - currents in $d-q$ system [A]
$u_{s d}, u_{S q}$ - voltages in $d-q$ system [V]
$n_{m}, f_{m}$ - mechanical speed [rpm], frequency $\left[\mathrm{Hz} ; \mathrm{s}^{-1}\right]$
$n_{e}, f_{e}$ - electrical speed [ $\mathrm{rpm}_{\mathrm{e}}$ ], frequency $\left[\mathrm{Hz}_{\mathrm{e}} ; \mathrm{s}_{\mathrm{e}}{ }^{-1}\right]$
$\omega_{m}$ - mechanical angular speed $\left[\mathrm{rad} \mathrm{s}^{-1}\right]$
$\omega_{e}$ - electrical angular speed $\left[\operatorname{rad}_{\mathrm{e}} \mathrm{s}^{-1}\right]$
$\vartheta_{m}$ - mechanical angle position [rad]
$\vartheta_{e}$ - electrical angle position $\left[\mathrm{rad}_{\mathrm{e}}\right]$
$\tau_{M}$ - motor driving torque $[\mathrm{Nm}]$
$\tau_{L} \quad$ - load torque $[\mathrm{Nm}]$

## B. Initial Physical Descriptrion

Let the system of the equations describing the physical basis of the PMSM begins by an equation of stator current equilibrium:

$$
\begin{equation*}
i_{S A}+i_{S B}+i_{S C}=0 \tag{1}
\end{equation*}
$$

and analogously by an equation of stator voltage equilibrium:

$$
\begin{equation*}
u_{S A}+u_{S B}+u_{S C}=0 \tag{2}
\end{equation*}
$$

Further crucial relation is the stator voltage distribution expressed by a set of the following equations:

$$
\begin{align*}
& u_{S A}=R_{S} i_{S A}+\frac{d}{d t} \psi_{S A} \quad u_{S A}=R_{S} i_{S A}+\frac{d}{d t}\left(L_{S} i_{S A}+\psi_{M A}\right)  \tag{3}\\
& u_{S B}=R_{S} i_{S B}+\frac{d}{d t} \psi_{S B} \rightarrow u_{S B}=R_{S} i_{S B}+\frac{d}{d t}\left(L_{S} i_{S B}+\psi_{M B}\right)  \tag{4}\\
& u_{S C}=R_{S} i_{S C}+\frac{d}{d t} \psi_{S C} \quad u_{S C}=R_{S} i_{S C}+\frac{d}{d t}\left(L_{S} i_{S C}+\psi_{M C}\right) \tag{5}
\end{align*}
$$

where each line belongs to appropriate individual phase. Equations (1) - (5) express the electro-magnetic properties of the stator coil winding (Fig. 2).


Figure 2. Pole permanent magnet fied windigs for 6 poles
The model in two-dimensional (2D) space of three-phase A-B-C system is completed by relation of electro-mechanical properties expressed by equation of torque equilibrium:

$$
\begin{align*}
J \ddot{\vartheta}_{M}=\sum_{i} \tau_{i} & \rightarrow J \dot{\omega}_{M}=\tau_{M}-B \omega_{M}-\tau_{L}  \tag{6}\\
& \rightarrow J \dot{\omega}_{e}=p \tau_{M}-B \omega_{e}-p \tau_{L}
\end{align*}
$$

where $\tau_{M}$ is a motor (driving) torque given by

$$
\begin{equation*}
\tau_{M}=\frac{p}{\omega_{e}}\left\{\frac{3}{2} \operatorname{Re}\left\{U_{s} I_{s}\right\}-3 R_{s} I_{s}^{2}\right\} \tag{7}
\end{equation*}
$$

$B \omega_{M}$ is a mechanical loss and $\tau_{L}$ is a load torque. All these quantities follow from the law of the energy conservation:

$$
\begin{gather*}
P_{\text {el. power input }}=P_{\text {mech.load }}+P_{\text {coil losses }}+P_{\text {losses in inron (mag.) }}+P_{\text {mech losses }} \\
\frac{3}{2} \operatorname{Re}\left\{U_{s} I_{s}\right\}=\tau_{L} \omega_{m}+3 R_{S} I_{S}^{2}+P_{F e}+B \omega_{m}^{2} \tag{8}
\end{gather*}
$$



Figure 3. 2D A-B-C and $\alpha-\beta$ coordinate systems

## C. Simplifiing Transformations

The equations (1) - (6) constitute the initial model representation in fixed 2D three-phase system for individual A, B, C phases. That model can be simplified both for simulation and for control design by two specific transformations.

The first is forward Clarke transformation (Fig. 3):

$$
\left[\begin{array}{l}
i_{S \alpha}  \tag{9}\\
i_{s \beta}
\end{array}\right]=k\left[\begin{array}{ccc}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{array}\right]\left[\begin{array}{l}
i_{S A} \\
i_{S B} \\
i_{S C}
\end{array}\right], \quad k=\frac{2}{3}
$$

Considering the current equilibrium (1), then the transformation can be reduced as follows

$$
\left[\begin{array}{c}
i_{S \alpha}  \tag{10}\\
i_{S \beta}
\end{array}\right]=k\left[\begin{array}{cc}
\frac{3}{2} & 0 \\
\frac{\sqrt{3}}{2} & \sqrt{3}
\end{array}\right]\left[\begin{array}{c}
i_{S A} \\
i_{S B}
\end{array}\right], \quad k=\frac{2}{3}
$$

This transformation converts (3) - (6) from 2D A-B-C phase system into 2D $\alpha-\beta$ system. The indicated transforming procedure is valid both current, voltage and flux components considering appropriate physical quantities respectively. It represents reduction of three phases or three appropriate phase axes in only two $\alpha-\beta$ axes. The axes are fixed to the stator coordinate system i.e. to the initial A-B-C phase system.

The transformed equations are expressed as follows:

$$
\begin{align*}
u_{S \alpha} & =R_{S} i_{S \alpha}+L_{S} \frac{d}{d t} i_{s \alpha}-\psi_{M} \sin \left(\vartheta_{e}\right) \dot{\vartheta}_{e}  \tag{11}\\
u_{S \beta} & =R_{S} i_{s \beta}+L_{S} \frac{d}{d t} i_{S \beta}+\psi_{M} \cos \left(\vartheta_{e}\right) \dot{\vartheta}_{e}  \tag{12}\\
J \ddot{\vartheta}_{e} & =\frac{3}{2} p^{2} \psi_{M}\left(\cos \vartheta_{e} i_{S \beta}-\sin \vartheta_{e} i_{S \alpha}\right)-B \omega_{e}-p \tau_{L} \tag{13}
\end{align*}
$$

The second transformation is forward Park transformation shown in Fig. 4:

$$
\left[\begin{array}{c}
i_{s d}  \tag{14}\\
i_{s q}
\end{array}\right]=\left[\begin{array}{cc}
\cos \vartheta_{e} & \sin \vartheta_{e} \\
-\sin \vartheta_{e} & \cos \vartheta_{e}
\end{array}\right]\left[\begin{array}{l}
i_{s \alpha} \\
i_{s \beta}
\end{array}\right]
$$



Figure 4. 2D $\alpha-\beta$ and $d-q$ coordinate systems
That transformation converts $2 \mathrm{D} \alpha-\beta$ system (11) - (13) into 2D $d-q$ system. The $d-q$ system unlike two fixed $\alpha-\beta$ axes is constituted by two rotating $d-q$ axes. The axes are connected to the rotating electromagnetic field of stator coil windings or rotating rotor with permanent magnets. AC PMSM is a synchronous motor as is mentioned directly in its label. Thus, the speed of electromagnetic rotating field is equal the speed of the rotor and proportionally synchronous with input current frequency.

The equations (11) - (13) applying (14) get the forms:

$$
\begin{align*}
u_{S d} & =R_{S} i_{S d}+L_{S} \frac{d}{d t} i_{S d}-L_{S} \omega_{e} i_{S q}  \tag{15}\\
u_{S q} & =R_{S} i_{S q}+L_{s} \frac{d}{d t} i_{S q}+L_{S} \omega_{e} i_{S d}+\psi_{M} \omega_{e}  \tag{16}\\
J \ddot{\vartheta_{e}} & =\frac{3}{2} p^{2} \psi_{M} i_{S q}-B \omega_{e}-p \tau_{L} \tag{17}
\end{align*}
$$

The $d$ - $q$ model (15) - (17) can be expressed in state-space like form (18):

$$
\begin{align*}
\frac{d}{d t}\left[\begin{array}{c}
i_{S d} \\
i_{S q} \\
\omega_{e} \\
\tau_{L}
\end{array}\right] & =\left[\begin{array}{cccc}
-\frac{R_{s}}{L_{s}} & 0 & 0 & 0 \\
0 & -\frac{R_{s}}{L_{s}} & -\frac{\psi_{\mu}}{L_{s}} & 0 \\
0 & \frac{3}{2} \frac{p^{2}}{J} \psi_{M} & -\frac{B}{J} & -\frac{p}{J} \\
0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
i_{S d} \\
i_{S q} \\
\omega_{e} \\
\tau_{L}
\end{array}\right]  \tag{18}\\
& +\left[\begin{array}{c}
\omega_{e} i_{S q} \\
-\omega_{e} i_{S d} \\
0 \\
0
\end{array}\right]+\left[\begin{array}{cc}
\frac{1}{L_{s}} & 0 \\
0 & \frac{1}{L_{s}} \\
0 & 0 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
u_{S d} \\
u_{S q}
\end{array}\right]
\end{align*}
$$

This model form represents as simple as possible mathe-matical-physical description suitable both for simulation and model-based control design. The model (18) contains nonlinear elements. They will be discussed in section IV. According to the indicated model forms and corresponding transformations in this section, the usual industrial control, i.e. cascade PI control, is structured as well. The brief description of the cascade control is given in the following section III. Finally, for further explanation, the full state vector $\left[i_{S d}, i_{S q}, \omega_{e}, \tau_{L}\right]^{T}$ is assumed to be known from measured variables ( $\left[i_{S A(B C)}, \omega_{e}, \tau_{L}\right]^{T}$ ) including also angular position $\vartheta_{e}$.


Figure 5. Speed control of PMSM by vector control (two-step cascade control)


Figure 6. Speed control of PMSM by Generalized Predictive Control

## III. Usual Cascade PI Control

As was mentioned, usual industrial control, i.e. cascade PI control, follows directly described way in section II. After measurement of individual phase currents and measurement or estimation rotor position and rotor speed, the currents are transformed stepwise by forward Clarke transformation and by forward Park transformation into $d-q$ coordinate system. In it, the main control operation is executed. The designed control actions ( $d-q$ voltages) are converted via inverse Park transformation back to $\alpha-\beta$ system ( $\alpha-\beta$ voltages). The control actions in $\alpha-\beta$ system are led to the Sinewave generator, which generates appropriate individual voltage magnitudes for individual A-B-C phases. The described way is illustrated in Fig. 5.

The illustrated scheme of speed control of PMSM consists of two interconnected loops. The main (master) loop is a speed loop. The subsidiary (slave) loop is current loop realized as two parallel legs corresponding to torque and flux control respectively. Each loop or leg contains isolated PI controller. From control theory point of view, this arrangement represents at least six control parameters (gains, time constants), which are usually empirically or by simple autotuning algorithm set up [8].

## IV. Model Modification and Assumptions for Predictive Control Design

As was mentioned, the suitable model for model-based control design is a model in $d-q$ coordinate system (18). In spite of its simplicity, it contains two nonlinear terms. Thus, for model based control, the model (18) has to be linearized, so that predictive control, a multistep approach, can be realized. The nonlinear terms may be linearized as follows:

$$
\left.\left[\begin{array}{c}
\omega_{e} i_{S q}  \tag{19}\\
-\omega_{e} i_{S d} \\
0 \\
0
\end{array}\right]=\left[\begin{array}{cccc}
0 & \omega_{e} & 0 & 0 \\
-\omega_{e} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{l}
i_{S d} \\
i_{S q} \\
\omega_{e} \\
\tau_{L}
\end{array}\right] \right\rvert\, \begin{aligned}
& i_{S d r}=0, i_{S q r}=0 \\
& \omega_{e r}=0, \tau_{L r}=0
\end{aligned}
$$

The linearization or linearizing decomposition (19) arises from the following idea [7] and specific reference state:

$$
\begin{align*}
\mathbf{f}_{(x, y, z)}= & \frac{\mathbf{f}_{(x, y, z)}-\mathbf{f}_{\left(x_{r}, y, z\right)}}{.\left(x-x_{r}\right)}\left(x-x_{r}\right)+\frac{\mathbf{f}_{\left(x_{r}, y, z\right)}-\mathbf{f}_{\left(x_{r}, y_{r}, z\right)}}{.\left(y-y_{r}\right)}\left(y-y_{r}\right)  \tag{20}\\
& +\frac{\mathbf{f}_{\left(x_{r}, y_{r}, z\right)}-\mathbf{f}_{\left(x_{r}, y_{r}, z_{r}\right)}}{.\left(z-z_{r}\right)}\left(z-z_{r}\right), \quad \text { if } \mathbf{f}_{\left(x_{r}, y_{r}, z_{r}\right)}=\mathbf{0}
\end{align*}
$$

Then, the resulting linearized form is:
$\frac{d}{d t}\left[\begin{array}{l}i_{S d} \\ i_{S q} \\ \omega_{e} \\ \tau_{L}\end{array}\right]=\left[\begin{array}{cccc}-\frac{R_{s}}{L_{S}} & \omega_{e} & 0 & 0 \\ -\omega_{e} & -\frac{R_{S}}{L_{s}} & -\frac{\psi_{\mu}}{L_{s}} & 0 \\ 0 & \frac{3}{2} \frac{p^{2}}{J} \psi_{M} & -\frac{B}{J} & -\frac{p}{J} \\ 0 & 0 & 0 & 0\end{array}\right]\left[\begin{array}{c}i_{S d} \\ i_{S q} \\ \omega_{e} \\ \tau_{L}\end{array}\right]+\left[\begin{array}{cc}\frac{1}{L_{S}} & 0 \\ 0 & \frac{1}{L_{s}} \\ 0 & 0 \\ 0 & 0\end{array}\right]\left[\begin{array}{l}u_{S d} \\ u_{S q}\end{array}\right](21)$
This model form represents already usual state-space model, but with time-variant terms:

$$
\begin{equation*}
\frac{d \mathbf{x}(t)}{d t}=\mathbf{A}_{C}(t) \mathbf{x}(t)+\mathbf{B}_{C} \mathbf{u}(t) \tag{22}
\end{equation*}
$$

$\mathbf{A}_{C}(t)$ is a time-variant state-space matrix, $\mathbf{B}_{C}$ is a constant input matrix. The variances of $\mathbf{A}_{C}(t)$ are given by variable $\omega_{e}$ elements, i.e. $\mathbf{A}_{c}(t)=\mathbf{A}_{C}\left(\omega_{e}(t)\right)$.

The model (21), as against (18), can be already discretized by standard exponential discretization procedure to the form:

$$
\begin{gather*}
\mathbf{x}_{k+1}=\mathbf{A}_{k} \mathbf{x}_{k}+\mathbf{B} \mathbf{u}_{k}  \tag{23}\\
\mathbf{y}_{k}=\mathbf{C} \mathbf{x}_{k} \tag{24}
\end{gather*}
$$

## V. Predictive Control

Predictive Control is based on a minimization of a quadratic criterion (25), in which the future system outputs are substituted by their predictions (26) expressed by the model given by (23) and (24) [1], [2]:

$$
\begin{align*}
& \min _{\mathbf{u}} J=\min _{\mathbf{u}} \mathbf{J}^{T} \mathbf{J}=\min _{\mathbf{u}}\left[\left\|\mathbf{Q}_{\mathbf{y}}(\hat{\mathbf{y}}-\mathbf{w})\right\|^{2}+\left\|\mathbf{Q}_{\mathbf{u}} \mathbf{u}\right\|^{2}\right]  \tag{25}\\
& \hat{\mathbf{y}}=\mathbf{f}+\mathbf{G u}, \mathbf{f}=\left[\begin{array}{c}
\mathbf{C A}_{k} \\
\vdots \\
\mathbf{C A}_{k}^{N}
\end{array}\right] \mathbf{x}_{k}, \mathbf{G}=\left[\begin{array}{ccc}
\mathbf{C B} & \cdots & \mathbf{0} \\
\vdots & \ddots & \vdots \\
\mathbf{C A}_{k}^{N-1} \mathbf{B} & \cdots & \mathbf{C B}
\end{array}\right] \tag{26}
\end{align*}
$$

where $\hat{\mathbf{y}}, \mathbf{w}$ and $\mathbf{u}$ are vectors of predictions (future predicted system outputs), references and control actions (system inputs) for a given prediction horizon $N$ :

$$
\hat{\mathbf{y}}=\left[\hat{\mathbf{y}}_{k+1}, \cdots, \hat{\mathbf{y}}_{k+N}\right]^{T}, \mathbf{w}=\left[\mathbf{w}_{k+1}, \cdots, \mathbf{w}_{k+N}\right]^{T}, \mathbf{u}=\left[\mathbf{u}_{k}, \cdots, \mathbf{u}_{k+N-1}\right]^{T}
$$

and $\mathbf{Q}_{\mathbf{y}}$ and $\mathbf{Q}_{\mathbf{u}}$ are weighting control parameters: output and input matrix penalizations. The predictions $\hat{\mathbf{y}}_{k+1}, \cdots, \hat{\mathbf{y}}_{k+N}$ in appropriate time instants of the prediction horizon can be expressed recurrently by using function (26).

The minimization of the criterion (25) can be provided in one shot as a least squares problem solution of algebraic system of equations [6]:

$$
\begin{align*}
& \mathbf{J}=\left[\begin{array}{cc}
\mathbf{Q} \mathbf{y} & \mathbf{0} \\
\mathbf{0} & \mathbf{Q u} \mathbf{u}
\end{array}\right]\left[\begin{array}{c}
\hat{\mathbf{y}}-\mathbf{w} \\
\mathbf{u}
\end{array}\right]=\underbrace{\left[\begin{array}{c}
\mathbf{Q} \mathbf{y} \mathbf{G} \\
\mathbf{Q} \mathbf{u}
\end{array}\right]}_{\mathbf{A}} \mathbf{\mathbf { u }}-\underbrace{\left[\begin{array}{c}
\mathbf{Q}(\mathbf{w}-\mathbf{f}) \\
\mathbf{0}
\end{array}\right]}_{\mathbf{b}}  \tag{27}\\
& \mathbf{J} \rightarrow \quad \min \Rightarrow \quad \mathbf{A} \mathbf{u}-\quad \mathbf{b} \quad=\mathbf{0} \\
& \mathbf{A u}=\mathbf{b} \quad / \mathbf{Q}^{T}  \tag{28}\\
& \mathbf{Q}^{T} \mathbf{A u}=\mathbf{Q}^{T} \mathbf{b}
\end{align*}
$$

where $\mathbf{Q}$ is orthogonal matrix, which rearranged matrix $\mathbf{A}$ into upper right triangle matrix $\mathbf{R}$ or $\mathbf{R}_{1}$ respectively as it is indicated:

$$
\begin{equation*}
\mathbf{R u}=\mathbf{c} \tag{29}
\end{equation*}
$$



Vector $\mathbf{c}_{\mathrm{z}}$ is a lost vector, whose Euclidean norm $\left|\mathbf{c}_{\mathrm{z}}\right|$ is equal value of square root $\sqrt{ }$ (i.e. $J=\mathbf{c}_{\mathrm{z}}{ }^{T} \mathbf{c}_{\mathrm{z}}$ ). To obtain unknown control actions $\mathbf{u}$, only upper part of the system (30) is need

$$
\begin{align*}
\mathbf{R}_{1} \mathbf{u} & =\mathbf{c}_{1} \\
\mathbf{u} & =\left(\mathbf{R}_{1}\right)^{T} \mathbf{c}_{1} \tag{31}
\end{align*}
$$

Since a matrix $\mathbf{R}_{1}$ is upper triangle, then the control $\mathbf{u}$ is given directly by back-run procedure.

## VI. Comparative Example

In this section, there is a brief description of one comparative example of the data from real experiment and the data obtained by simulation. The real experiment was realized on Siemens PMSM drive 1FK7022-5AK-1LG0 [8].

In the Fig. 7, there is time history of real measured data from the real experiment. In the Fig. 8, there is time history of the simulation data. The comparative simulation is provided by the mathematical model derived in section II. The used model parameters of PMSM drive were taken from configuration manual [8] for the same motor as mentioned above.

The figures show similar curses of time histories of corresponding physical quantities: mechanical speed $\omega_{m}$, phase voltages $u_{S A(B C)}$ and phase currents $i_{S A(B C)}$. The obvious smoothness of the simulation is caused by considering the motor as ideal system without any disturbances. The both experiments run for triangular profile of desired rotational speed values within the interval <-100rpm, $100 \mathrm{rpm}>$. The condition on zero (minimum) currents was included in both cases: real experiment and simulation.


Figure 7. Speed control of PMSM by two-step cascade PI control - time histories of real experiment, sampling period $\mathrm{Ts}=0.000125 \mathrm{~s}$


Figure 8. Speed control of PMSM by Generalized Predictive Control - time histories of simulation; horizon $N=8$, sampling period Ts $=0.000125 \mathrm{~s}$

## VII. CONCLUSION

The paper deals with a study of Predictive Control design for PMSM drives. The general model of the PMSM was explained and used in model-based control design. The industrial cascade PI control was briefly explained as well. The comparative example demonstrates the correspondence of industrial realization and model-based design approach realization. Predictive Control seems to be promising way to optimize the drive control with considering other different requirements. The requirements, to be considered, may be e.g. requirements on different types of drive constraints, which cannot be solved via conventional control systems.

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